A *quadratic function* is a polynomial of degree two.
The *standard form* of a quadratic function is

$$f(x) = ax^2 + bx + c,$$

where $a$, $b$, and $c$ are real numbers. The graph is a *parabola* which opens upward if $a > 0$ and opens downward if $a < 0$. The $y$-intercept is the point $(0, f(0))$, and we see that $f(0) = c$. The roots of the function are the values of $x$ such that $f(x) = 0$. The *quadratic formula* says that $f(x) = 0$ if and only if

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

The *discriminant* of the quadratic function is

$$\Delta = b^2 - 4ac;$$

this determines the number of real roots. There are three cases:

(a) if $b^2 - 4ac > 0$, there are two real roots;

(b) if $b^2 - 4ac = 0$, there is one real root;

(c) if $b^2 - 4ac < 0$, there are no real roots.

The $x$-intercepts (if any) are the points $(x, 0)$, where $x$ is a real root.
The *shifted form* of a quadratic function is

$$f(x) = a(x - h)^2 + k,$$

where $a$, $h$, and $k$ are real numbers. The shifted form tells how the graph of $f(x)$ is obtained from the graph of $x^2$, as follows:

(a) shift horizontally by $h$;

(b) stretch vertically by $|a|$;

(c) reflect across the $x$-axis if $a$ is negative;

(d) shift vertically by $k$.

The point $(h, k)$ where the graph turns around is called the *vertex*. Thus $k$ is the *minimum value* of the function if $a > 0$, and is the *maximum value* of the function is $a < 0$.
We can convert from standard form to shifted form by completing the square, which leads to:
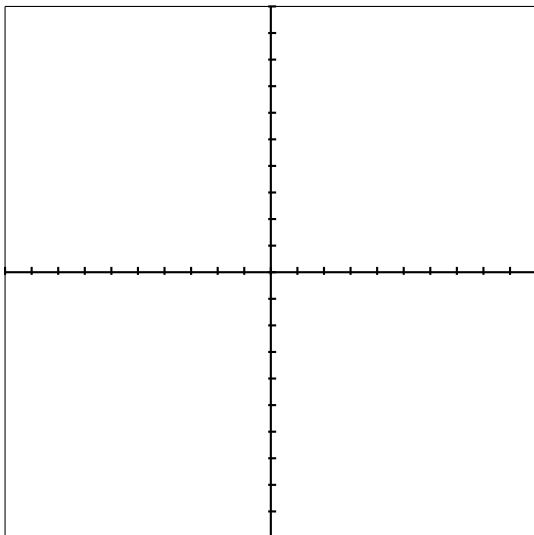
$$h = -\frac{b}{2a} \quad \text{and} \quad k = c - \frac{b^2}{4a}.$$

We can convert from shifted form to standard form by squaring and simplifying, which leads to:
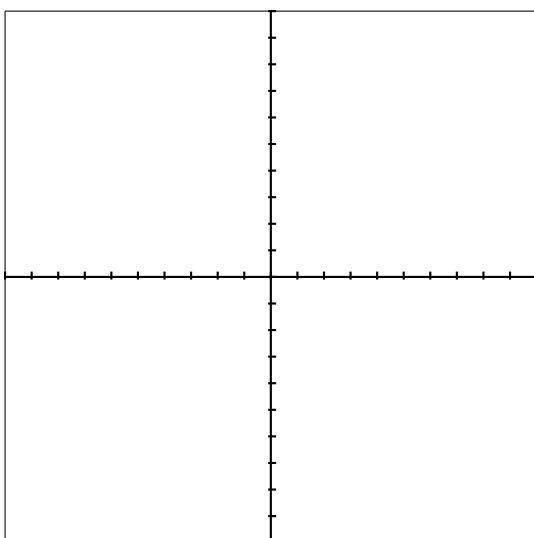
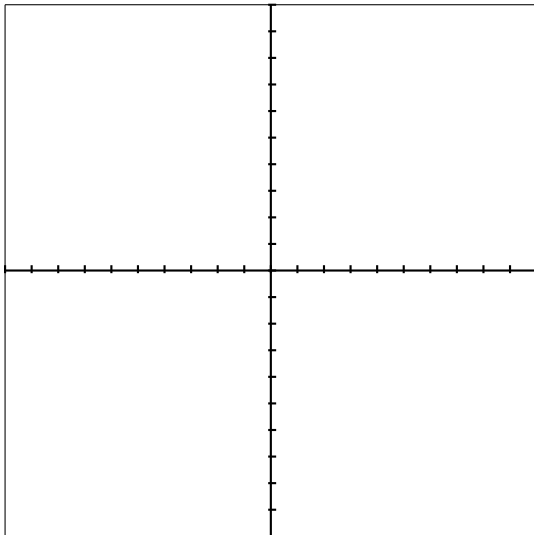$$b = -2ah \quad \text{and} \quad c = ah^2 + k.$$
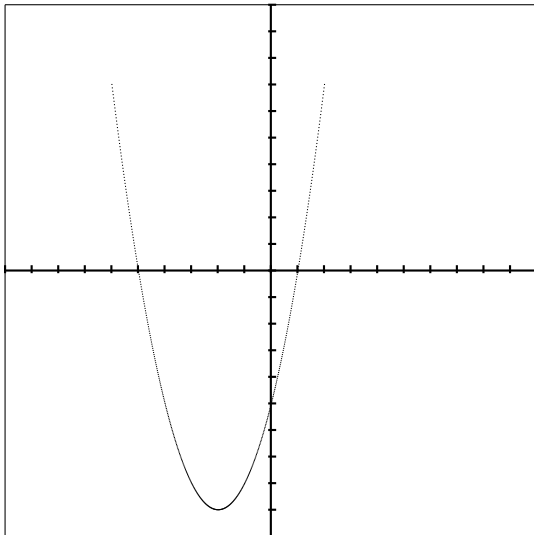
**Example:** $f(x) = 4x - 5 + x^2$

**Standard Form:** $x^2 + 4x - 5$

**Shifted Form:** $(x + 2)^2 - 9$

**a:** 1 **b:** 4 **c:** $-5$ **h:** $-2$ **k:** $-9$

**Discriminant:** 36

**Roots:** $x = -5$ and $x = 1$

**$y$-intercept:** $(0, -5)$

**$x$-intercept(s):** $(-5, 0)$ and $(1, 0)$

**Vertex:** $(-2, -9)$

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**Problem 1:** $f(x) = x^2 - 2x - 8$

**Standard Form:**

**Shifted Form:**

**a:** **b:** **c:** **h:** **k:**

**Discriminant:**

**Roots:**

**$y$-intercept:**

**$x$-intercept(s):**

**Vertex:**

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**Problem 2:** $f(x) = (x + 2)^2 - 7$

**Standard Form:**

**Shifted Form:**

**a:** **b:** **c:** **h:** **k:**

**Discriminant:**

**Roots:**

**$y$-intercept:**

**$x$-intercept(s):**

**Vertex:**

**Problem 3:** $f(x) = 5x - x^2$

**Standard Form:**

**Shifted Form:**

**a:**      **b:**      **c:**         **h:**         **k:**
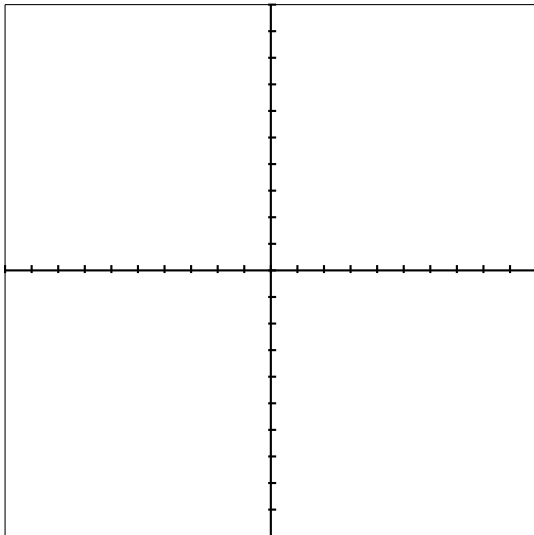
**Discriminant:**

**Roots:**

**y-intercept:**

**x-intercept(s):**

**Vertex:**

**Problem 4:** $f(x) = (3x - 7)(-x + 1)$

**Standard Form:**

**Shifted Form:**
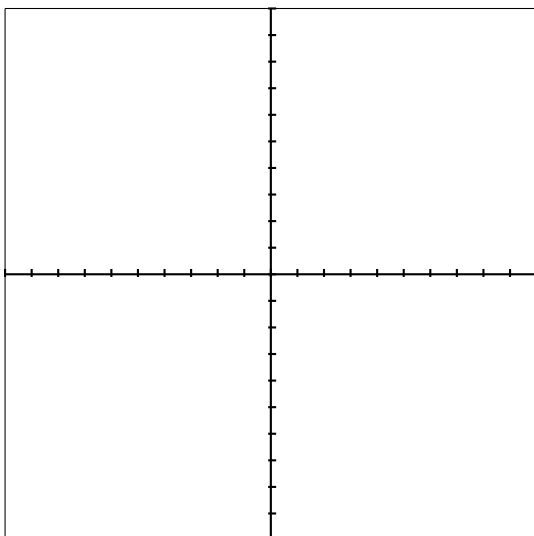
**a:**      **b:**      **c:**         **h:**         **k:**

**Discriminant:**

**Roots:**

**y-intercept:**

**x-intercept(s):**

**Vertex:**

**Problem 5:** $f(x) = 6 + x^2 - 4x$

**Standard Form:**

**Shifted Form:**

**a:**      **b:**      **c:**         **h:**         **k:**

**Discriminant:**

**Roots:**

**y-intercept:**

**x-intercept(s):**

**Vertex:**